

Vertex Switching behaviour of Divided Square Difference Cordial Graphs

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Abstract: In this article, we discuss the vertex switching behaviour of some divided square difference (DSD) cordial graphs such as path graph, cycle graph, star graph, wheel graph, helm graph, bistar graph and gear graph.

Keywords: vertex switching, path graph, cycle graph, star graph, wheel graph, helm graph, bistar graph, gear graph.

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I. INTRODUCTION

By a graph, we mean a finite, undirected graph without loops and multiple edges. Graph labeling has large number of applications in Mathematics as well as in several areas of Computer science and Communication networks. A. Alfred Leo et.al introduced the concept of divided square difference cordial labeling graphs [1]. Also they discussed DSD cordial labeling of some special graphs in [2]. They introduced the balanced DSD cordial labelling in [3]. The concept of cordial labeling was introduced by Cahit [4]. S.N.Daoud discussed edge odd graceful labeling for gear graph in [5]. The motivation behind DSD cordial labeling is due to R. Dhavaseelan et.al [6]. A dynamic survey on different graph labeling along with an extensive bibliography was found in Gallian [7]. For standard terms we refer to Harary [8]. V.J.Kaneria et.al [9] introduced the concept of balanced cordial labeling. In 1967, Rosa [10] introduced a labeling of G called β -valuation. R.Varatharajan, et.al [11] have introduced the notion of divisor cordial labeling. The motivation behind this work is due to S.K.Vaidya et.al [12] who discussed E-Cordial Labeling in the context of switching of a vertex. R.Vikramaprasad et.al discussed sum corial labeling for wheel graph, helm graph, bistar graph in [13].

In this article, we discuss the vertex switching behaviour of some divided square difference (DSD) cordial graphs such as path graph, cycle graph, star graph, wheel graph, helm graph, bistar graph and gear graph.

II. PRELIMINARIES

Definition 2.1 [7]

The *Graph labeling* is an assignment of numbers to the edges or vertices or both subject to certain condition(s). If the domain of the mapping is the set of vertices (edges), then the labeling is called a vertex (edge) labeling.

Definition 2.2 [7]

A *vertex labeling* of a graph G is an assignment f of labels to the vertices of G that induces each edge uv a label depending on the vertex label $f(u)$ and $f(v)$.

Definition 2.3 [7]

A mapping $f: V(G) \rightarrow \{0,1\}$ is called *binary vertex labeling* of G and $f(v)$ is called the label of the vertex v under f .

Definition 2.4 [4]

A binary vertex labeling f of a graph G is called a *Cordial labeling* if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is cordial if it admits cordial labeling.

Definition 2.5 [8]

The wheel graph W_n is a graph formed by connecting a single universal vertex to all vertices of a cycle.

Definition 2.6 [13]

The helm graph H_n is the graph obtained from the wheel graph W_n by adjoining a pendent edge at each node of the cycle.

Definition 2.7 [13]

Bistar is the graph obtained by joining apex vertices of two copies of star $K_{1,n}$.

Definition 2.8 [5]

The gear graph G_n is obtained from the wheel graph W_n by adding a vertex between every pair of adjacent vertices of the cycle.

Definition 2.9 [12]

A vertex switching G_v of a graph G is the graph obtained by taking a vertex v of G and removing the entire edges incident to v and adding the edges joining v to every other vertices which are not adjacent to v in G .

Definition 2.10 [9]

A cordial graph G with a cordial labeling f is called a *balanced cordial graph* if $|e_f(0) - e_f(1)| = |v_f(0) - v_f(1)| = 0$.

It is said to be *edge balanced cordial graph* if $|e_f(0) - e_f(1)| = 0$ and $|v_f(0) - v_f(1)| = 1$. Similarly it is said to be *vertex balanced cordial graph* if $|e_f(0) - e_f(1)| = 1$ and $|v_f(0) - v_f(1)| = 0$. A cordial graph G is said to be *unbalanced cordial graph* if $|e_f(0) - e_f(1)| = |v_f(0) - v_f(1)| = 1$.

Definition 2.11 [1]

Let $G = (V, E)$ be a simple graph and $f: V \rightarrow \{1, 2, 3, \dots, |V|\}$ be a bijection. For each edge uv , assign the label 1 if $\left| \frac{(f(u))^2 - (f(v))^2}{f(u) - f(v)} \right|$ is odd and the label 0 otherwise. f is called *divided square difference (DSD) cordial labeling* if $|e_f(0) - e_f(1)| \leq 1$, where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively.

A graph G is called *divided square difference (DSD) cordial* if it admits divided square difference cordial labeling.

Definition 2.12 [3]

A divided square difference cordial graph G is called a *balanced DSD cordial graph* if $|e_f(0) - e_f(1)| = 0$ and *unbalanced DSD cordial graph* if $|e_f(0) - e_f(1)| = 1$.

Proposition 2.13 [1]

1. Any path P_n is a DSD cordial graph.
2. Any cycle C_n is a DSD cordial graph except $n \equiv 2 \pmod{4}$.
3. The star graph $K_{1,n}$ is DSD cordial.

Proposition 2.14 [2]

1. The wheel graph W_n , $n \equiv 0, 1 \pmod{4}$ is DSD cordial.
2. The helm graph H_n , $n \equiv 0, 1 \pmod{4}$ is DSD cordial.
3. Bistar $B_{m,m}$ is a DSD cordial graph.

Proposition 2.15 [3]

The gear graph G_n is a balanced DSD cordial when n is even.

III. RESULTS AND DISCUSSION

Proposition 3.1

Any DSD cordial path P_n is a balanced DSD cordial graph when n is odd.

Proof

By Proposition 2.13, let G be a DSD cordial path P_n with $|V(G)| = n$ and $|E(G)| = n - 1$.

In this graph, $|e_f(0) - e_f(1)| = \begin{cases} 0, & n \text{ is odd} \\ 1, & n \text{ is even} \end{cases}$

Thus P_n is a balanced DSD cordial graph when n is odd and unbalanced DSD cordial graph when n is even.

Proposition 3.2

The graph obtained by switching of any vertex other than the pendent vertex in a DSD cordial path P_n ($n \geq 4$) is a DSD cordial graph.

Proof

Let G be a path graph P_n ($n \geq 4$) with $|V(G)| = n$ and $|E(G)| = n - 1$. Let v_1, v_2, \dots, v_n are the vertices of the path P_n . By Proposition 2.13, we draw a DSD cordial path P_n . Now, we can construct the new graph G_v by switching any arbitrary vertex v_i ($1 < i < n$) other than the pendent vertex. In this graph G_v , $|V(G)| = n$ and $|E(G)| = 2n - 6$. Then by labeling the graph G_v using definition 2.11, we get $|e_f(0) - e_f(1)| \leq 1$.

Hence G_v is also a divided square difference cordial graph.

Remark 3.3

From Proposition 3.2, in particular we get $|e_f(0) - e_f(1)| = 0$.

Hence from Propositions 3.1 and 3.2, we can conclude that G is a balanced DSD cordial graph when n is odd whereas G_v is a balanced DSD cordial when n is odd as well as even.

Example 3.4

Figure 1(a) and 1(b) illustrates the unbalanced DSD cordial graph G (P_8) and balanced DSD cordial graph G_v obtained by switching the vertex v_2 respectively.

Figure 2(a) and 2(b) illustrates the balanced DSD cordial graph G (P_7) and balanced DSD cordial graph G_v obtained by switching the vertex v_2 respectively.

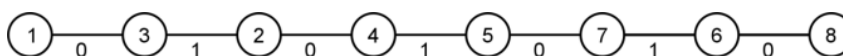


Figure 1(a)

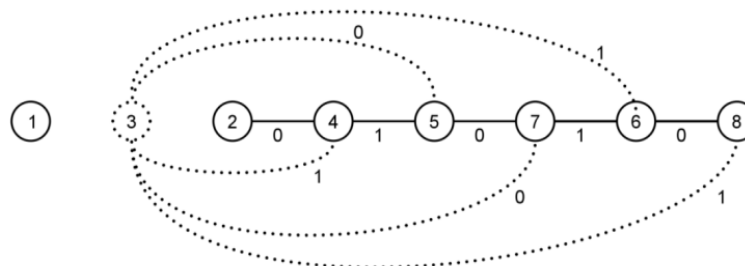


Figure 1(b)

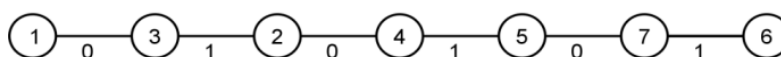


Figure 2(a)

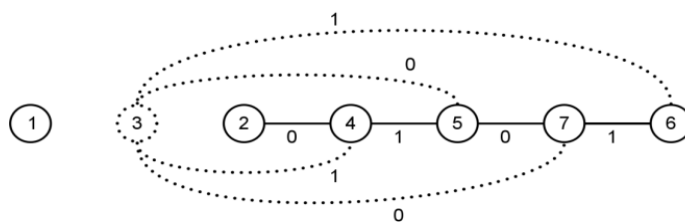


Figure 2(b)

Proposition 3.5

Any DSD cordial cycle C_n is a balanced DSD cordial graph when n is even.

Proof

By Proposition 2.13, let G be a DSD cordial cycle graph C_n with $|V(G)| = n$ and $|E(G)| = n$.

$$|e_f(0) - e_f(1)| = \begin{cases} 0, & n \text{ is even} \\ 1, & n \text{ is odd} \end{cases}$$

Thus, C_n is a balanced DSD cordial graph when n is even and unbalanced DSD cordial graph when n is odd.

Proposition 3.6

The graph obtained by switching of any arbitrary vertex in a DSD cordial cycle C_n is DSD cordial.

Proof

Let G be a cycle graph C_n with $|V(G)| = n$ and $|E(G)| = n$. Let v_1, v_2, \dots, v_n are the vertices of the cycle C_n . By Proposition 2.13, we draw a DSD cordial cycle graph C_n . Now, we can construct the new graph G_v by switching any arbitrary vertex $v_i (1 \leq i \leq n)$. In this graph $G_v, |V(G)| = n$ and $|E(G)| = 2n - 5$. Then by labeling the graph G_v using definition 2.11, we get $|e_f(0) - e_f(1)| \leq 1$.

Hence G_v is also a divided square difference cordial graph.

Remark 3.7

From Proposition 3.6, in particular we get $|e_f(0) - e_f(1)| = 1$.

Hence from Propositions 3.5 and 3.6, we can conclude that G is a balanced DSD cordial graph when n is even whereas G_v is an unbalanced DSD cordial when n is odd as well as even.

Example 3.8

Figure 3(a) and 3(b) illustrates the balanced DSD cordial graph $G (C_8)$ and unbalanced DSD cordial graph G_v obtained by switching the vertex v_1 respectively.

Figure 4(a) and 4(b) illustrates the unbalanced DSD cordial graph $G (C_9)$ and unbalanced DSD cordial graph G_v obtained by switching the vertex v_2 respectively.

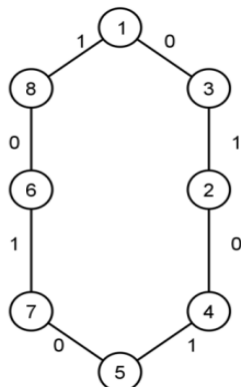


Figure 3(a)

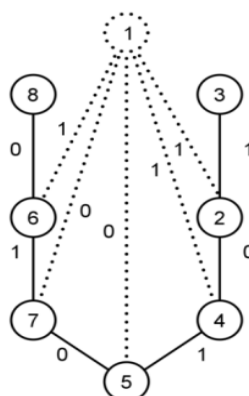


Figure 3(b)

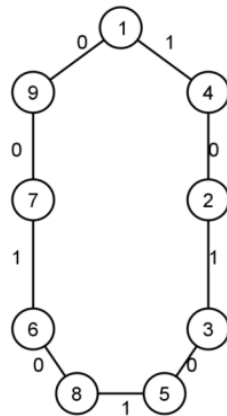


Figure 4(a)

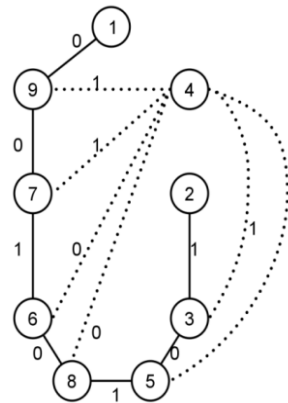


Figure 4(b)

Proposition 3.9

The DSD cordial star graph $K_{1,n}$ is a balanced DSD cordial when n is even.

Proof

By Proposition 2.13, let G be a DSD cordial star graph $K_{1,n}$ with $|V(G)| = n + 1$ and $|E(G)| = n$.

In this graph, $|e_f(0) - e_f(1)| = \begin{cases} 0, n \text{ is even} \\ 1, n \text{ is odd} \end{cases}$.

Thus, $K_{1,n}$ is a balanced and unbalanced DSD cordial graph when n is even and odd respectively.

Proposition 3.10

The graph obtained by switching of any vertex $v_i (i = 1,3,5 \dots, n)$ in a DSD cordial graph $K_{1,n}$ is DSD cordial.

Proof

Let G be a star graph $K_{1,n}$ with $|V(G)| = n + 1$ and $|E(G)| = n$. Let v be the central vertex and v_1, v_2, \dots, v_n are the end vertices of the star $K_{1,n}$.

By Proposition 2.13, we draw a DSD cordial star graph $K_{1,n}$. Now, we can construct a new graph G_v by switching any vertex $v_i (i = 1,3,5 \dots)$.

In this graph G_v , $|V(G)| = n + 1$ and $|E(G)| = 2n - 2$.

Then by labeling the graph G_v using definition 2.11, we get $|e_f(0) - e_f(1)| \leq 1$.

Hence G_v is also a divided square difference cordial graph.

Remark 3.11

From Proposition 3.10, in particular we get $|e_f(0) - e_f(1)| = 0$.

Hence from Propositions 3.9 and 3.10, we can conclude that G is a balanced DSD cordial graph when n is even whereas G_v is a balanced DSD cordial for all n .

Example 3.12

Figure 5(a) and 5(b) illustrates the unbalanced DSD cordial graph $G(K_{1,7})$ and balanced DSD cordial graph G_v obtained by switching the vertex v_3 respectively.

Figure 6(a) and 6(b) illustrates the unbalanced DSD cordial graph $G(K_{1,8})$ and balanced DSD cordial graph G_v obtained by switching the vertex v_5 respectively.

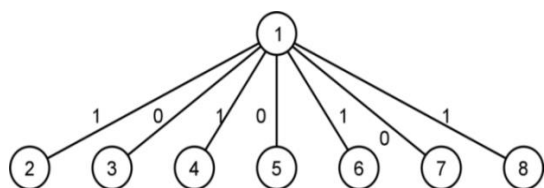


Figure 5(a)

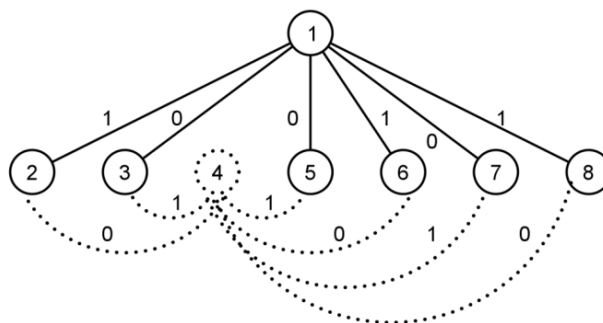


Figure 5(b)

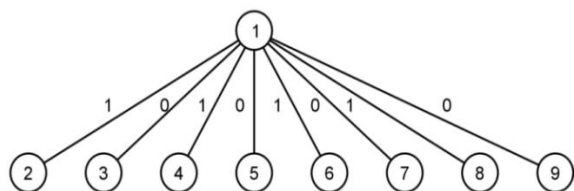


Figure 6(a)

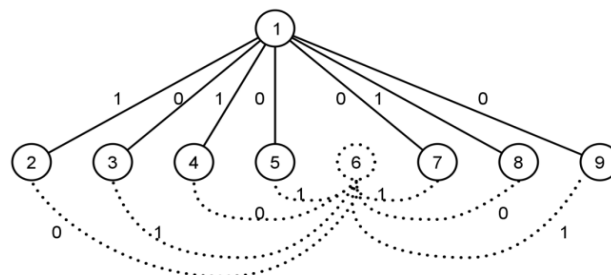


Figure 6(b)

Proposition 3.13

The DSD cordial wheel graph W_n ($n \equiv 0,1 \pmod{4}$) is a balanced DSD cordial.

Proof

By Proposition 2.14, let G be a DSD cordial wheel graph W_n with $|V(G)| = n + 1$ and $|E(G)| = 2n$.

In this graph, $|e_f(0) - e_f(1)| = 0$.

Thus, W_n is a balanced divided square difference cordial graph.

Proposition 3.14

The graph obtained by switching of any rim vertex $v_i(1 \leq i \leq n)$ in a DSD cordial wheel graph W_n is DSD cordial.

Proof

Let G be a wheel graph W_n with $|V(G)| = n + 1$ and $|E(G)| = 2n$. Let u be the central vertex and v_1, v_2, \dots, v_n are the rim vertices of wheel W_n . By proposition 2.14, we draw a DSD cordial wheel graph W_n .

We will discuss two cases.

Case i: When n is odd

Construct the new graph G_v by switching any odd labeled rim vertex $v_i(1 \leq i \leq n - 1)$. In this graph G_v , $|V(G)| = n + 1$ and $|E(G)| = 3n - 6$. Then by labeling the graph G_v using definition 2.11, we get $|e_f(0) - e_f(1)| \leq 1$.

Hence G_v is also a divided square difference cordial graph.

Case ii: When n is even

Construct the new graph G_v by switching any even labeled rim vertex $v_i(1 \leq i \leq n)$. Then by labeling the graph G_v using definition 2.11, we get $|e_f(0) - e_f(1)| \leq 1$.

Hence G_v is also a divided square difference cordial graph.

Remark 3.15

From Proposition 3.14, in particular we get $|e_f(0) - e_f(1)| = 0$ when $n \equiv 0 \pmod{4}$ and $|e_f(0) - e_f(1)| = 1$ when $n \equiv 1 \pmod{4}$.

Hence from Propositions 3.13 and 3.14 we can conclude that G is a balanced DSD cordial graph whereas G_v is a balanced DSD cordial when $n \equiv 0 \pmod{4}$ and unbalanced DSD cordial when $n \equiv 1 \pmod{4}$.

Example 3.16

Figure 7(a) and 7(b) illustrates the balanced DSD cordial graph $G(W_9)$ and unbalanced DSD cordial graph G_v obtained by switching the vertex v_4 respectively.

Figure 8(a) and 8(b) illustrates the balanced DSD cordial graph $G(W_8)$ and balanced DSD cordial graph G_v obtained by switching the vertex v_8 respectively.

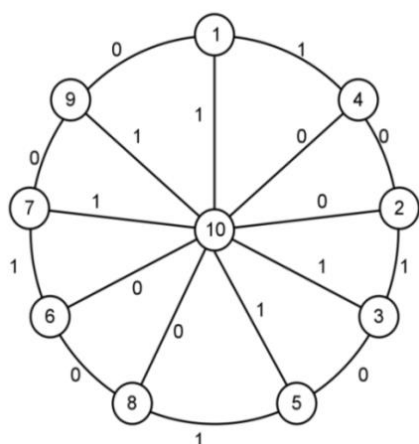


Figure 7(a)

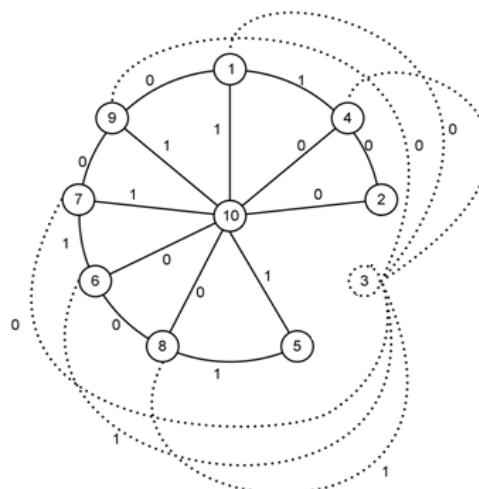


Figure 7(b)

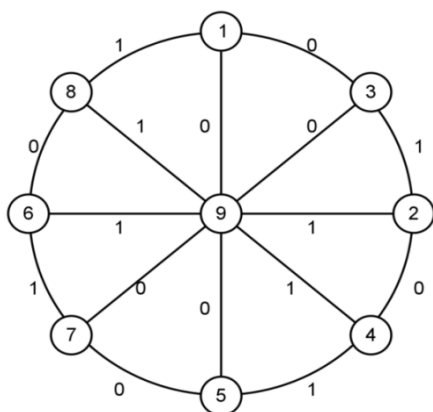


Figure 8(a)

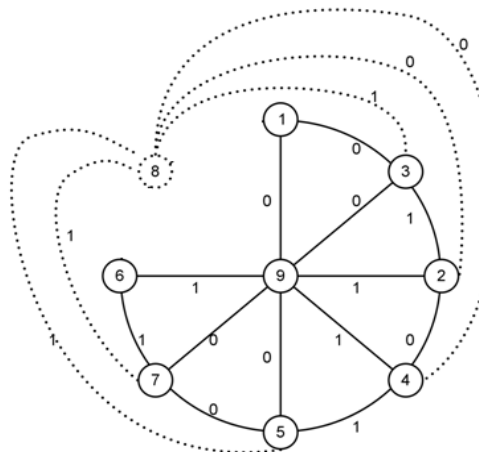


Figure 8(b)

Proposition 3.17

The helm graph H_n ($n \equiv 0,1 \pmod{4}$) is a balanced DSD cordial graph when n is even.

Proof

By Proposition 2.14, let G be a DSD cordial Helm graph H_n with $|V(G)| = 2n + 1$ and $|E(G)| = 3n$.

In this graph, $|e_f(0) - e_f(1)| = \begin{cases} 0, n \text{ is even} \\ 1, n \text{ is odd} \end{cases}$.

Thus, H_n is a balanced divided square difference cordial graph when n is even and unbalanced DSD cordial when n is odd.

Proposition 3.18

The graph obtained by switching of an apex vertex x in a DSD cordial graph H_n is DSD cordial.

Proof

Let G be a helm graph H_n with $|V(G)| = 2n + 1$ and $|E(G)| = 3n$. Let $x, v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n$ are the vertices of H_n . Here x is the apex vertex, v_1, v_2, \dots, v_n are the vertices of the cycle C_n and u_1, u_2, \dots, u_n are the pendent vertices. By Proposition 2.14, we draw a DSD cordial helm graph H_n . Now, we can construct the new graph G_v by switching the apex vertex x . In this graph G_v , $|V(G)| = 2n + 1$ and $|E(G)| = 3n$. Then by labeling the graph G_v using definition 2.11, we get $|e_f(0) - e_f(1)| \leq 1$.

Hence G_v is also a divided square difference cordial graph.

Remark 3.19

From Proposition 3.18, in particular we get

$$|e_f(0) - e_f(1)| = \begin{cases} 0, & n \text{ is even} \\ 1, & n \text{ is odd} \end{cases}$$

Hence from Propositions 3.17 and 3.18, we can conclude that the DSD cordial graphs G and G_v are balanced DSD cordial when n is even and unbalanced DSD cordial when n is odd.

Example 3.20

Figure 9(a) and 9(b) illustrates the balanced DSD cordial graph $G(H_8)$ and balanced DSD cordial graph G_v obtained by switching the apex vertex respectively.

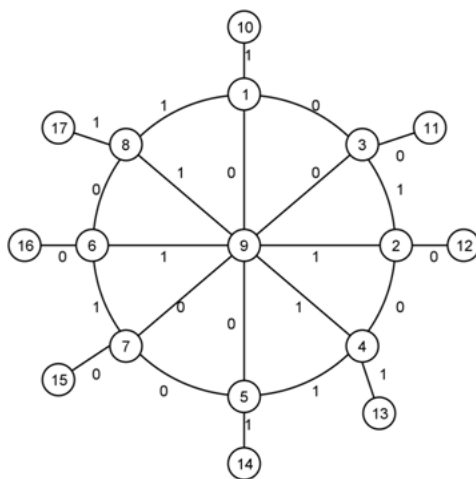


Figure 9(a)

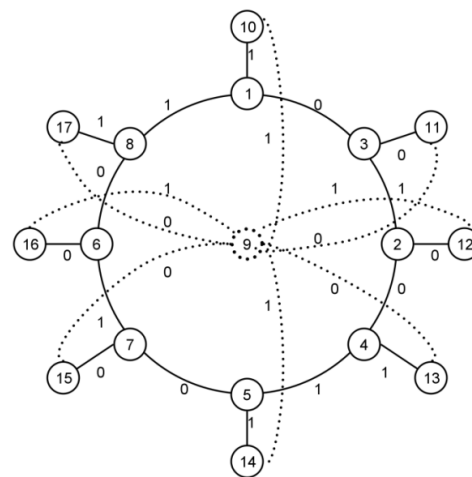


Figure 9(b)

Proposition 3.21

Bistar $B_{m,m}$ is a unbalanced DSD cordial graph.

Proof

By Proposition 2.14, let G be a DSD cordial bistar graph $B_{m,m}$ with $|V(G)| = 2m + 2$ and $|E(G)| = 2m + 1$. In this graph, $|e_f(0) - e_f(1)| = 1 \forall m$.

Thus, bistar $B_{m,m} (\forall m)$ is an unbalanced divided square difference cordial graph.

Proposition 3.22

The graph obtained by switching of an apex vertex in a DSD cordial graph $B_{m,m}$ is a DSD cordial graph when m is even.

Proof

Let G be a bistar graph $B_{m,m}$ with $|V(G)| = 2m + 2$ and $|E(G)| = 2m + 1$.

Let u_1, u_2, \dots, u_m and v_1, v_2, \dots, v_m are the vertices of each copy of $K_{1,m}$ with the apex vertex x and y . Let the edge set be $E(G) = \{xy, yv_i, xu_i, 1 \leq i \leq m\}$. By Proposition 2.14, we draw a DSD cordial bistar graph $B_{m,m}$ where m is even. Now, we can construct the new graph G_v by switching any of the apex vertex in $B_{m,m}$. In this graph G_v , $|V(G)| = 2m + 2$ and $|E(G)| = 2m$. Then by labeling the graph G_v using definition 2.11, we get $|e_f(0) - e_f(1)| \leq 1$.

Hence G_v is also a divided square difference cordial graph.

Remark 3.23

From Proposition 3.22, in particular we get $|e_f(0) - e_f(1)| = 0$.

Hence from Propositions 3.21 and 3.22, we can conclude that G is a unbalanced DSD cordial graph whereas the graph G_v is a balanced DSD cordial.

Example 3.24

Figure 10(a) and 10(b) illustrates the unbalanced DSD cordial graph $G(B_{6,6})$ and balanced DSD cordial graph G_v obtained by switching the apex vertex x respectively.

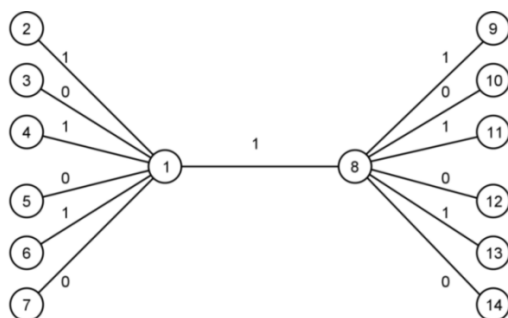


Figure 10(a)

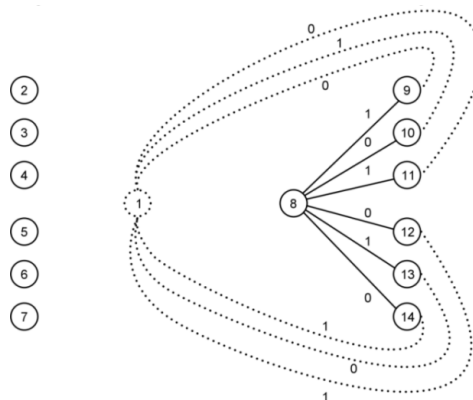


Figure 10(b)

Proposition 3.25

The graph obtained by switching the apex vertex in a DSD cordial gear graph G_n is DSD cordial.

Proof

Let G be a gear graph G_n with $|V(G)| = 2n + 1$ and $|E(G)| = 3n$. Let v_1, v_2, \dots, v_n are the vertices of the cycle C_n and x be the apex vertex. By Proposition 2.15, we draw a DSD cordial gear graph G_n . Now, we can construct the new graph G_v by switching the apex vertex w in G_n . In this graph G_v , $|V(G)| = 2n + 1$ and $|E(G)| = 3n$. Then by labeling the graph G_v using definition 2.11, we get $|e_f(0) - e_f(1)| \leq 1$.

Hence G_v is also a divided square difference cordial graph.

Remark 3.26

From Proposition 3.25, in particular we get $|e_f(0) - e_f(1)| = \begin{cases} 0, & n \text{ is even} \\ 1, & n \text{ is odd} \end{cases}$

Hence, we can conclude that the graphs G and G_v are balanced DSD cordial when n is even and unbalanced DSD cordial when n is odd.

Example 3.27

Figure 11(a) and 11(b) illustrates the unbalanced DSD cordial graph $G(G_7)$ and unbalanced DSD cordial graph G_v obtained by switching the apex vertex respectively.

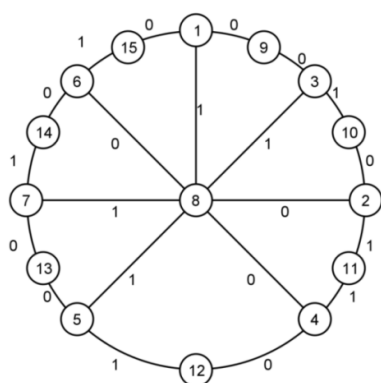


Figure 11(a)

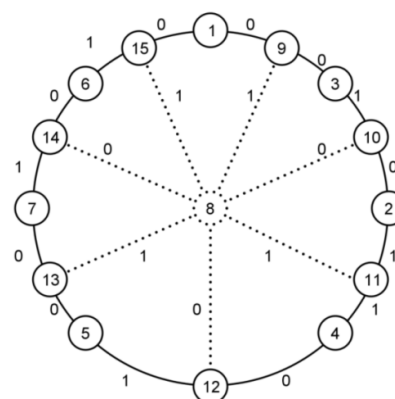


Figure 11(b)

IV. CONCLUSION

In this article, we have proved that the graph obtained by switching a vertex in the DSD cordial graphs such as path graph P_n , cycle graph C_n , star graph $K_{1,n}$, wheel graph W_n , helm graph H_n , bistar graph $B_{m,m}$ and gear graph G_n is DSD cordial.

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